Progressive Education Society's

Seat No...........

Modern College of Arts, Science and Commerce (Autonomous)

Shivajinagar, Pune -5

[Total no. of questions:4] [Total number of pages:2]

First Year B.Sc. Computer Science (Mar-2020)

End Semester Backlog Examination, (2019 Pattern) Semester – I

Course Code: 19CsMatU101 **Course Name: Discrete Mathematics**

Date: 16-03-2020 **Time: 10.00 a.m. - 12.00 p.m.**

[Time: 2 Hours] [Max Marks: 60]

N.B. :- (i) All questions are compulsory.
(ii) Figures to the right indicate full marks.
(iii) Neat diagrams must be drawn wherever necessary.

Q 1. Attempt any *five* of the following : [$2 \times 5 = 10$]

1. Write contrapositive and the converse of the following statement :
"If it is raining then the home team wins".

2. Determine the truth set of the following proposition over positive integers :
(a) $p(n) = n$ is a perfect square and $n < 100$
(b) $q(n) = n$ is prime and $n < 25$.

3. State Idempotent and Absorption laws in a lattice.

4. State first principle of Mathematical Induction.

5. Show that if 7 colours are used to paint 50 cars, at least 8 cars will have the same colour.

6. Find first four terms of the sequence defined by the following recurrence relation :
$a_n = a_{n-1} + 2\, a_{n-2};\ a_0 = 1,\ a_1 = 2.$

7. Find characteristic roots of the recurrence relation $a_n - 6\, a_{n-1} + 9\, a_{n-2} = 0$.

Q 2. Attempt any *three* of the following : [$3 \times 5 = 15$]

1. Define Universal quantifier and Existential quantifier.
Let $\phi(x, y)$: "$x + y = 0$" and $U = \mathbb{R}$. Write truth values of the following with justification :
(a) $\exists y\ \forall x\ \phi(x, y)$
(b) $\forall x\ \exists y\ \phi(x, y)$.

2. Show the following logical equivalence using truth table :
$[a \rightarrow (b \wedge c)] \equiv [(a \rightarrow b) \wedge (a \rightarrow c)]$

3. Draw Hasse diagram of Lattice D_{20}. Is it a complemented lattice? Justify.

4. If the join operation is distributive over the meet operation in a lattice, then prove that the meet operation is also distributive over the join operation.

Q 3. Attempt any *three* of the following : $[3 \times 5 = 15]$

1. Using combinatorial argument, prove that :

   $$C(n, r) + C(n, r - 1) = C(n + 1, r).$$

2. In a survey of 100 people, it is found that 78 like oranges, 47 like mangoes and 10 like neither. How many people like both ? How many like oranges but not mangoes ?

3. Solve the following recurrence relation :

   $$a_n = 4\, a_{n-1} - 4\, a_{n-2},$$

   with initial conditions $a_0 = 6,\ a_1 = 8$

4. Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time. How many ways can this person climb a flight of eight stairs ?

Q 4. Attempt any *two* of the following : $[2 \times 10 = 20]$

1. (a) Give direct proof for :

   $$p,\ p \rightarrow q,\ s \vee r,\ r \rightarrow \sim q \ \vdash s \vee t.$$ (5)

   (b) Give indirect proof for :

   $$\sim p \vee q,\ s \vee p,\ \sim q \ \vdash \ s.$$ (5)

2. Find disjunctive normal form of the following Boolean expression :

   $$E(x, y, z) = (x \wedge \overline{(y \vee z)}) \vee (y \wedge z).$$

3. (a) In how many ways can the letters in the following word be arranged ?

   "COMPUTER" (4)

   (b) How many numbers are there between 100 and 1000 in which all the digits are distinct ? (6)

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