



Modern College of Arts, Science and Commerce (Autonomous)

Shivajinagar, Pune -5

[Total no. of questions:04]

[Total number of pages:02]

First Year B.Sc. Computer Science (Mar-2020)

End Semester Backlog Examination, (2019 Pattern) Semester – I

Course Code: 19CsMatU102

Course Name: Algebra

Date: 17-03-2020

Time: 10.00 a.m. to 12.00 p.m.

[Time: 2 Hours]

[Max Marks: 60]

- N.B. 1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1) Attempt any five of the following:

[5x2=10]

- On \mathbb{Z} , the set of integers, define $*$ as $a * b = a^b$. Justify whether $*$ is binary operation or not.
- Let G be a group with identity element e . Show that if $a^2 = e$ for all $a \in G$, then G is an abelian group.
- Let a, b, c be integers. If $a|b$ and $a|c$ then $a|b + c$.
- Find the elements (if any) in \mathbb{Z}_6 that satisfy $x^2 = x$.
- Find the value of $(-\bar{3} +_{11} \bar{5})$.
- Reduce the following matrix to row echelon form.

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$$

- Write the solution set for the system $x + y - 2z - w = 0$.

Q.2) Attempt any three of the following:

[3x5=15]

- Solve the following system of linear equations by Gaussian Elimination method

$$\begin{aligned} 2x + 2y + 2z &= 0 \\ -2x + 5y + 2z &= 1 \\ 8x + y + 4z &= -1 \end{aligned}$$

- Prove that integers 361 and 420 are relatively prime.
- If $a \equiv b \pmod{n}$, $c \equiv d \pmod{n}$ then prove that $(a + c) \equiv (b + d) \pmod{n}$.
- Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 1 & 6 & 5 \end{pmatrix}$$
$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 3 & 6 \end{pmatrix}$$

be permutations in S_6 .

- σ and τ is even or odd.
- Find $O(\sigma)$, $O(\tau)$
- Express σ as a product of disjoint cycles.

Q.3) Attempt **any three** of the following:

[3x5=15]

a) Find rank of the following matrix A .

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 4 & 3 & 1 & 5 \\ 0 & -1 & 3 & 2 \\ -1 & 5 & 1 & 0 \end{bmatrix}$$

b) Solve the following system by **LU decomposition method**.

$$\begin{aligned} -3x_1 + 12x_2 - 6x_3 &= -33 \\ x_1 - 2x_2 + 2x_3 &= 7 \\ x_2 + x_3 &= -1 \end{aligned}$$

c) Write addition and multiplication tables of residue classes *modulo* 7 .Hence find $(\bar{6})^{-1}$ with respect to \times_7 and $(\bar{3})^{-1}, (\bar{4})^{-1}$ with respect to $+_7$.

d) Find remainder when 17^{1402} is divided by 13.

Q.4)Attempt **any two** of the following:

[10x2=20]

a) Define $*$ on Q^+ (set of positive rational numbers) as $a * b = \frac{ab}{7}$.Then show that $(Q^+, *)$ is an abelian group.

b) Find GCD of 7677 and 4647 . Also , find integers m and n such that $7677 m + 4647 n = gcd(7677, 4647)$.

c) For which values of λ does the following system of equations have
(i) no solutions (ii) exactly one solution (iii) infinitely many solutions.

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (\lambda^2 - 14)z &= \lambda + 2 \end{aligned}$$