Progressive Education Society's

Modern College of Arts, Science and Commerce Shivajinagar (Autonomous), Pune 411005

Department of Mathematics

S.Y.B.Sc.

Ordinary Differential Equations

QUESTION BANK

Subject Teacher: Prof. Pooja M. Paratane

Question: Attempt the following: (2 Marks Questions)

1. Test the linear independence of the following sets of functions
   (a) $1+x, 1+2x, x^2$
   (b) $x^2-1, x^2-x+1, 3x^2-x-1$
   (c) $e^x, e^{-x}, \sin ax$
   (d) $\sin x, \cos x, \sin 2x$
2. Prove that $\sin 2x$ and $\cos 2x$ are solutions od differential equation $y''+4y=0$ and these solutions are linearly independent.
3. Find the general solution of differential equation $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$
4. Let $f(D)=3D^2-2D+1$. Calculate $f(D)(e^{2x})$.
5. Show that $x=2e^{4t}$, $y=3e^{4t}$ and $x=e^{-t}$, $y=-e^{-t}$ are solutions of the homogeneous system $\frac{dx}{dt}=x+2y$, $\frac{dy}{dt}=3x+2y$.
6. Find the general solution of $y''+y=0$.
7. Show that $x=e^{3t}$, $y=e^{3t}$ and $x=e^{2t}$, $y=2e^{2t}$ are solutions of the homogeneous system $\frac{dx}{dt}=4x-y$, $\frac{dy}{dt}=2x+y$.
8. If $f(D)=D+2$ and $g(D)=3D-1$ are differential operators, then find the differential operator $g(D)f(D)$.
9. Determine the singular points of the differential equation $(1-x^2)y''-6xy'-4y=0$.
10. Locate and classify singular points of $x^2y''+(2-x)y'=0$.
11. Find the Wronskian of $f_1(x)=x^2$ and $f_2(x)=x^3$ for all $x\in\mathbb{R}$.
12. Determine the intervals in $\mathbb{R}$ in which the differential equation $\tan x\frac{d^2y}{dx^2}+y=0$ is normal.
13. Locate and classify singular points of $2x^2y''+x(2x+1)y'-y=0$.
14. Convert the differential equation $y''+y'=0$ into a system of first order differential equations.
15. Determine the intervals in $\mathbb{R}$ in which the differential equation $(1-x^2)\frac{dy}{dx}+\sin x\ y=\cos x$ is normal.

16. Verify that origin is a regular singular point of the differential equation $2x^2y'' + 2y' + y = 0$.
17. Verify that $y_1 = x$ is a solution of $(1-x^2)y'' - 2xy' + 2y = 0$.
18. Find the particular solution of the differential equation $(D-2)^2 y = e^{2x}$.
19. Convert the differential equation $y'' + 2y' - y = e^x$ into a system of first order differential equations.
20. Convert the differential equation $y''' - y = 0$ into a system of first order differential equations.
21. Show that $f_1(x) = e^x$ and $f_2(x) = e^{-x}$ are linearly independent on $[-1, 1]$.
22. Verify that $y = x^2$ is one solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$
23. Show that $f_1(x) = \sin x$ and $f_2(x) = \cos x$ are linearly independent on $[0, \pi]$.
24. Determine the intervals in $\mathbb{R}$ in which the differential equation $x^2\frac{d^2y}{dx^2} + e^x y = \log x$ is normal.

Question: Attempt the following: (5 Marks Questions)

1. Solve the linear differential equation $x\frac{dy}{dx} + y = x^4$.
2. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^x$ using the method of reduction of order.
3. Find the power series solution of the differential equation $yâĂš + y = 0$.
4. Find the power series solution of the differential equation $yâĂš' + y = 0$.
5. Solve the following system of first order differential equations $\frac{dx}{dt} = 3x + 3y$, $\frac{dy}{dt} = -x - y$.
6. Solve the linear differential equation $\left(D^2 - 2D + 1\right)y = xe^x + 7x - 2$.
7. Solve the differential equation $\left(D^2 + 5D + 6\right)y = e^{-2x}\sec^2 x(1 + 2\tan x)$.
8. Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ using the method of variation of parameters.
9. Find power series solution of the differential equation $y' + 4y = 0$.
10. Solve the differential equation $\left(D^2 - 3D + 2\right)y = e^{2x} + 5$.
11. Find the general solution of $(D^2 + 4)y = \sin 3x - \cos x$.
12. Solve the differential equation $xy'' - y' = 3x^2$ by using method of reduction of order.
13. Solve $\frac{d^2y}{dx^2} + y = cosecx$ using the method of variation of parameters.
14. Find power series solution of the differential equation $y'' - 4y = 0$.
15. Solve the differential equations by method of variation of parameters $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \frac{1}{x^2}$.

16. Find the general solution of $(D^2+3D+2)y = e^x \cos 2x$.
17. Find the general solution of $(D^4-1)y = \cos x$.
18. Solve the differential equation $(D^2-2D+1)y = x^2e^{2x}$.
19. Solve $(D^2+4)y = x\sin 2x$.
20. Solve the differential equation $\frac{d^2y}{dx^2} - (1+x)\frac{dy}{dx} + xy = x$by using method of reduction of order.
21. Solve the differential equation $(D^5 - D^3)y = 1$.
22. Solve the differential equations by method of variation of parameters $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$.
23. Find power series solution of the differential equation $y'' - y = 0$.
24. Solve the differential equation $(D^2+2D+1)y = \frac{1}{(e^x-1)^2}$ by using method of reduction of order.
25. Solve $(D^2+1)y = 2$ by using variation of parameters method.
26. Find the power series solution of the differential equation $y' + 9y = 0$.
27. Verify that $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$, then find y_2 and the general solution.
28. Solve the differential equation $x^2y'' + y' - (1+x^2)y = e^{-x}$ by using method of reduction of order.
29. Solve the differential equation $(D^2+9)y = \cos 3x$

Question: Attempt the following: (10 Marks Questions)

1. Solve the following system of first order differential equations $x' = 4x + y$; $y' = -4x + 8y$
2. Find the power series solution of the differential equation $y'' - y' + xy = 0$.
3. Solve the following system of first order differential equations $x' = 2x - 5y$; $y' = 2x - 4y$
4. Find the power series solution of the differential equation $y'' - xy = 0$.
5. Find the general solution of $(D^2-2D+1)y = xe^x \sin x$.
6. Solve the system of differential equations $\frac{dx}{dt} = 4x + 5y$, $\frac{dy}{dt} = -4x + 4y$.
7. Find the power series solution of the differential equation $y'' + x^2y = 0$.
8. Solve the system of differential equations $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 4x - 2y$.
9. Find the power series solution of the differential equation $(1-x^2)y'' - 6xy' - 4y = 0$ at the ordinary point $x = 0$.
10. Solve the system of differential equations $\frac{dx}{dt} + 2x + 4y = 1 + 4t$, $\frac{dy}{dt} - 3x + 2y = \frac{3t^2}{2}$.
11. Find power series solution of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

12. Solve the system of differential equations $4\frac{dx}{dt}-\frac{dy}{dt}+3x=\sin t,\ \frac{dx}{dt}+y=\cos t.$

13. Show that $x=0$ is an ordinary point of differential equation $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$. Hence, find the power series solution of it.

14. Find the power series solution of the differential equation $(x^2-1)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0$ near $x=0$.

15. Solve the system of differential equations $\frac{dx}{dt}-\frac{dy}{dt}-y=-e^t,\ \frac{dy}{dt}+x-y=e^{2t}.$

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