Progressive Education Society's
Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune 411005

Department of Mathematics

T.Y.B.Sc. (Mathematics)

Operations Research

Practical 1: Formulation of LPP and Graphical Method

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1. Wild West produces two types of cowboy hats. A type 1 hat requires twice as much labor time as a type 2. If all the available time is dedicated to Type 2 alone, the company can produce a total of 400 type 2 hats per day. The respective market limits for type 1 and type 2 are 150 and 200 hats per day, respectively. The profit is Rs.8 per type 1 hat and Rs.5 per type 2 hat. Determine the number of hats of each type that maximizes profit. (Solve Graphically)

2. Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. The following table provides the basic data of the problem. (Solve Graphically)

Raw Material	**Tons of material per ton of**		**Maximum daily availability (tons)**
	Exterior Paint	**Interior paint**	
M1	6	4	24
M2	1	2	6
Profit per ton (Rs.1000)	5	4	

   A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also the maximum daily demand for interior paint is 2 tons. Formulate the LPP.

3. Old hens can be bought at Rs. 20 each and young ones at Rs. 50each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week; each egg being worth 1 rupee. A hen (young or old) costs Rs. 2 per week to feed. I have only Rs. 800 to spend for hens. How many of each kind should I buy to get a maximum profit assuming that I cannot house more than 20 hens? (Solve Graphically)

4. A firm manufactures two types of products A and B, sells it at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines M1 and M2. Type A requires 1 minute processing on M1 and 2 minutes on M2. Now Type requires 1 minute processing on M1 and 1 minute on M2. The machine M1 is available for not more than 6 hours 40 minutes while M2 is available for 10 hours during any working day. Formulate LPP. (Solve Graphically)

5. Chemlabs produces two domestic cleaning solutions A and B by processing two raw materials I and II. The processing of 1 unit of raw material I costs Rs. 80 and produces 0.5 unit of solution A and 0.5 unit of solution B. Moreover the processing of 1 unit of raw material II costs Rs. 50 and produces 0.4 unit of solution A and produces 0.6 unit of solution B. The daily demand for solution A lies between 10 and 15 units and that for solution B lies between 12 and 20 units. Formulate LPP. (Solve Graphically)

6. An assembly line consisting of three consecutive stations produces two radio models; HiFi-1 and HiFi-2. The following table provides the assembly times for three workstations:

	Minutes per unit	
Workstation	HiFi-1	HiFi-2
W1	6	4
W2	5	5
W3	4	6

The daily maintenance for solutions W1, W2, W3 consumes 10%, 14% and 12% respectively of the maximum 480 units available for each station each day. Formulate the LPP that will minimize the idle (or unused) times in the three workstations. (Solve Graphically)

7. Solve the following LPP graphically :

$$\begin{aligned}
&\text{Minimize } \ Z = 3x_1 + 5x_2 \\
&\text{subject to,} \\
&-3x_1 + 4x_2 \leq 12 \\
&2x_1 + 3x_2 \geq 12 \\
&2x_1 - x_2 \geq -2 \\
&0 \leq x_1 \leq 4 \\
&x_2 \geq 2
\end{aligned}$$

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Practical 2: Simplex and Big-M Method

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1. Solve the following LPP by Simplex Method:

$$\begin{aligned}&\text{Maximize } Z = x_1 + 4x_2 + 5x_3\\&\text{subject to,}\\&3x_1 + 3x_2 \leq 22\\&x_1 + 2x_2 + 2x_3 \leq 14\\&3x_1 + 2x_2 \leq 14\\&x_1,\ x_2,\ x_3 \geq 0\end{aligned}$$

2. Determine whether the following LPP has alternate solution, if yes, obtain it:

$$\begin{aligned}&\text{Maximize } Z = x_1 + 2x_2\\&\text{subject to,}\\&-x_1 + 2x_2 \leq 8\\&x_1 + 2x_2 \leq 12\\&x_1 - 2x_2 \leq 3\\&x_1,\ x_2 \geq 0\end{aligned}$$

3. Solve the following LPP by Big-M method:

$$\begin{aligned}&\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3\\&\text{subject to,}\\&3x_1 + x_2 + 4x_3 \leq 600\\&2x_1 + 4x_2 + 2x_3 \geq 480\\&2x_1 + 3x_2 + 3x_3 = 540\\&x_1,\ x_2,\ x_3 \geq 0\end{aligned}$$

4. Show that the following LPP has unbounded solution:

$$\begin{aligned}&\text{Maximize } Z = 20x_1 + 10x_2 + x_3\\&\text{subject to,}\\&3x_1 - 3x_2 + 5x_3 \leq 50\\&x_1 + x_3 \leq 10\\&x_1 - x_2 + 4x_3 \leq 20\\&x_1,\ x_2,\ x_3 \geq 0\end{aligned}$$

5. Solve the following LPP:

$$\begin{aligned}
&\text{Minimize } \ Z = 2x_1 + x_2 \\
&\text{subject to,} \\
&1.5x_1 + x_2 \leq 6 \\
&x_1 \leq 2 \\
&x_1 + x_2 \geq 7 \\
&-x_1 + x_2 \geq 4 \\
&x_1,\ x_2,\ x_3 \geq 0
\end{aligned}$$

6. Solve the following LPP by Big-M method:

$$\begin{aligned}
&\text{Minimize } \ Z = 60x_1 + 80x_2 \\
&\text{subject to,} \\
&20x_1 + 30x_2 \geq 900 \\
&40x_1 + 30x_2 \geq 1200 \\
&x_1,\ x_2, \geq 0
\end{aligned}$$

7. Solve the following linear programming problem by Big-M method.

$$\begin{aligned}
&\text{Minimize } \ Z = 2x_1 - 3x_2 + 6x_3 \\
&\text{subject to,} \\
&3x_1 - 4x_2 - 6x_3 \leq 2 \\
&2x_1 + x_2 + 2x_3 \geq 11 \\
&x_1 + 3x_2 - 2x_3 \leq 5 \\
&x_1,\ x_2, x - 3 \geq 0
\end{aligned}$$

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Practical 3: Duality in LPP

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1. Consider the following LPP:

$$\begin{aligned}&\text{Maximize } Z = 2x_1 + 3x_2\\&\text{subject to,}\\&x_1 + 3x_2 \leq 6\\&3x_1 + 2x_2 \leq 6\\&x_1,\ x_2 \geq 0\end{aligned}$$

   (a) Express the problem in standard form
   (b) Determine all the basic solutions of the problem and classify them as feasible and infeasible.
   (c) Use direct substitution in the objective function to determine the optimum basic feasible solution.
   (d) Verify graphically that the solution obtained in (c) is optimal solution.
   (e) Show how the infeasible basic solutions are represented on the graphical solution space

2. Find all basic solutions to the following problem

$$\begin{aligned}&\text{Maximize } Z = x_1 + x_2 + 3x_3\\&\text{subject to,}\\&x_1 + 2x_2 + 3x_3 = 4\\&3x_1 + 2x_2 \leq 6\end{aligned}$$

   Also, Find which of the basic solutions are

   i) Basic feasible solution
   ii) Non-degenerate basic feasible solution
   iii) Optimal solution

3. Solve the dual of the following problem. Then, find optimal solution of primal from the solution of dual.

$$\begin{aligned}&\text{Minimize } Z = 3x_1 + 2x_2\\&\text{subject to,}\\&2x_1 + x_2 \geq 4\\&x_1 + 7x_2 \geq 7\\&x_1,\ x_2 \geq 0\end{aligned}$$

4. Find the dual of the following LPP , solve it by simplex method and obtain the solution of primal LPP:

$$\begin{aligned}&\text{Minimize } Z = 4x_1 + 3x_2 + x_3\\&\text{subject to,}\\&x_1 + 2x_2 + 4x_3 \geq 12\\&3x_1 + 2x_2 + x_3 \geq 8\\&x_1,\ x_2,\ x_3 \geq 0\end{aligned}$$

5. Find the dual of the following LPP , solve it by simplex method and obtain the solution of primal LPP:

Maximize $Z = 2x_1 + x_2$
subject to,
$x_1 + x_2 \geq 2$
$x_1 + 3x_2 \leq 3$
$x_1,\ x_2 \geq 0$

6. Write the dual of the following LPP, solve it by simplex method and obtain the solution of primal.

Maximize $Z = x_1 + 5x_2$
subject to,
$x_1 + 2x_2 \leq 3$
$2x_1 - x_2 \geq 4$
$x_1,\ x_2 \geq 0$

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Practical 4: Transportation Problem

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1. Find IBFS of the following transportation problem using:
   (a) North West corner Rule
   (b) Least Cost Entry Method

Warehouse	**Stores**				**Availability**
	I	II	III	IV	
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	

2. Find IBFS of the following transportation problem using VAM method

Source	**Destination**				**Supply**
	D1	D2	D3	D4	
S1	15	18	22	16	30
S2	15	19	20	14	40
S3	13	16	23	17	30
Demand	20	20	25	35	

3. Find IBFS of following transportation problem using LCEM method and hence obtain optimal solution using MODI method.

Source	**Destination**				**Supply**
	D1	D2	D3	D4	
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

4. Obtain initial basic feasible solution of the following transportation problem using VAM. Determine whether the solution is optimal, if not find the optimal solution.

	1	**2**	**3**	**4**	**Capacity**
P_1	19	30	50	12	7
P_2	70	30	40	60	10
P_3	40	10	60	20	18
Demand	5	8	7	15	

5. Consider the following transportation problem with IBFS.

	A	B	C	D	Supply
1	5	10	4 (10)	5	10
2	6 (20)	8	7	2 (5)	25
3	4 (5)	2 (10)	5 (5)	7	20
Demand	25	10	15	5	

Answer the following questions:

(a) Is the solution feasible?

(b) Is this solution degenerate?

(c) Is this solution optimal?

(d) If not, obtain optimal solution.

6. Find the optimum solution of the following transportation problem. Is the solution unique? If not find alternate solution.

Factory	**Sales Agency**			
	I	II	III	IV
P1	7	5 (10)	6	4
P2	3	5	4	2 (15)
P3	4 (8)	6	4 (12)	5
P4	8	7 (2)	6 (6)	5 (7)

7. Obtain the optimal solution of the following maximization transportation problem.

	A	B	C	D	E	
1	80	90	60	75	100	80
2	55	65	40	60	50	60
3	30	25	30	50	20	75
4	15	12	25	35	10	80
	75	60	50	80	100	

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Practical 5: Assignment Problem

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1. Consider a problem of assigning four clerks to four tasks. The time(hours) required to complete the tasks is given below.

		Tasks			
		A	B	C	D
clerks	1	4	7	5	6
	2	-	8	7	4
	3	3	-	5	3
	4	6	6	4	2

2. Suggest the optimum assignment to sales territories, where estimates of sales to be made by each salesman in different territories are given below.

		Territories				
		I	II	III	IV	V
Salesman	A	10	15	17	14	14
	B	6	18	10	12	16
	C	12	5	13	13	6
	D	8	11	16	10	12

3. A computer center has got 4 expert programmer. The center need 4 programs to be developed. The head of the computer center after carefully studying the programs estimated the time in minutes required by respective experts programmers as follows:

		PROGRAMES			
		A	B	C	D
EXPERTS	1	120	100	80	90
	2	80	90	110	70
	3	110	140	120	100
	4	90	90	80	90

4. Solve the following assignment problem.

		Workers				
		W1	W2	W3	W4	W5
Jobs	A	15	10	25	25	10
	B	1	8	10	20	2
	C	8	9	17	20	10
	D	14	10	25	27	15
	E	10	8	25	27	12

5. Find optimal solution of the following assignment problem. Also, find the alternate optimal solution, if it exists.

	I	**II**	**III**	**IV**	**V**
A	3	9	2	3	7
B	6	1	5	6	6
C	9	4	7	10	3
D	9	6	2	4	5
E	2	5	4	2	1

6. A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1,2,3,4,5 and 6. The distance between the cities in kilometres is shown in the matrix below. Find the assignment of trucks from cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum.

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

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Practical 6: Miscellaneous

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1. A farmer has 1,000 acres of land on which he can grow corn, wheat or soyabeans. Each acre of corn costs Rs.100/- for preparation, requires 7 man-days of work and yields a profit of Rs.30/-. An acre of wheat costs Rs.120/- to prepare, requires 10 man-days of work and yields a profit of Rs.40. An acre of soyabeans costs Rs.70/- to prepare, requires 8 man-days of work and yields a profit of Rs.20/-. If the farmer has Rs.1,00,000/- for preparation and can count on 8,000 man-days of work. Formulate linear programming model to allocate the number of acres to each crop to maximize the total profit.

2. Find the initial basic feasible solution of the following transportation problem by Least Cost Entry method and obtain its optimal solution by MODI method.

	W_1	W_2	W_3	W_4	**Supply**
O_1	2	3	4	5	6
O_2	5	4	3	1	8
O_3	1	3	3	2	10
Demand	4	6	8	6	

3. Use Big-M method to solve the following linear programming problem.

Minimize $Z = 2x_1 - 3x_2 + 6x_3$
subject to,
$3x_1 - 4x_2 - 6x_3 \leq 2$
$2x_1 + x_2 + 2x_3 \geq 11$
$x_1 + 3x_2 - 2x_3 \leq 5$
$x_1, x_2, x_3 \geq 0$

4. Solve the following assignment problem of maximization.

	A	**B**	**C**	**D**	**E**
I	32	38	40	28	40
II	40	24	28	21	36
III	41	27	33	30	37
IV	22	38	41	36	36
V	29	33	40	35	39

5. The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the inputs and outputs per production run are as follows:

Process	**Input(units)**		**Output(units)**	
	Crude A	**Crude B**	**Gasoline X**	**Gasoline Y**
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crudes A and B is 200 units and 150 units respectively. Market requirement show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 300/- and Rs. 400/- respectively. Formulate the linear programming problem to maximize the profit.

6. Solve the dual of the following problem. Then, find optimal solution of primal from the solution of dual.

$$\begin{aligned}&\text{Minimize } \ Z = 3x_1 + 2x_2\\ &\text{subject to,}\\ &2x_1 + x_2 \geq 4\\ &x_1 + 7x_2 \geq 7\\ &x_1,\ x_2, \geq 0\end{aligned}$$

7. Popeye Canning is contracted to receive 60,000 Kg. of ripe tomatoes at Rs. 7per Kg. from which it produces both canned juice and tomato paste. The canned products are packaged in 24 can cases. A can of juice requires 1 Kg. of fresh tomatoes and a can of paste requires 1/3 Kg. only. The companys share of the market is limited to 2000 cases of juice and 6000 cases of paste. The wholesale prices of per case of juice and paste are Rs. 1800 and Rs. 900 resp. Formulate LPP for maximum revenue. Solve Graphically.

8. Find IBFS of following transportation problem using VAM method and hence obtain optimal solution using MODI method.

Plant	**Warehouse**				**Supply**
	W1	W2	W3	W4	
P1	6	4	9	1	40
P2	20	6	11	3	40
P3	7	1	0	14	50
P4	7	1	12	6	90
Demand	90	30	50	30	