*Progressive Education Society's*

Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune 411005

**Department of Mathematics**

**S.Y.B.Sc.**

Numerical Method and its Applications

**Assignment 1: Errors**

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1. Round-off the following numbers correct to 4-significant figures
   a) 20.56235 b) 0.00024357 c) 2689.5487
2. An approximate value of $\pi$ is given by 3.14278152 and its value is 3.14159265. Find the absolute and relative errors.
3. Find the relative error of the number $\frac{5}{7}$ whose approximate value is 0.714.
4. Round-off the numbers 395.367 and 0.013526 to four significant digits and calculate the relative errors.
5. The approximate values of the number $\frac{1}{6}$ are given as $0.165, 0.166$ and 0.167. Which of these three are best approximation?
6. Round-off the number 0.987250 correct to four significant figures and find percentage error.
7. An approximate value of $e$ is 2.7195518 and it's true value is given by $x = 2.71821828$. Find the relative error.
8. Find the relative error of the number 11.426.

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Numerical Methods and its Applications

**Assignment 2: Solutions of Algebraic and Transcendental Equations**

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1. Obtain Newton Raphson formula to find $r^{th}$ root of given number $c$ and hence find $\sqrt[4]{14}$.
2. Use Regula- Falsi Method to find real root of equation $e^{-x} - 4x = 0$. (4 iterations)
3. Find the approximate root of the equation $e^x \cos x = 1.4$ using Newton Raphson Method.
4. Use the iterative method to find, correct to four significant figures of the equation
   (a) $\sin x = 10(x-1)$
   (b) $x = \dfrac{1}{(x+1)^2}$
5. Find $\sqrt[3]{28}$ by constructing an equation. Apply Newton Raphson Method.
6. Estimate the positive root of $x^2 - \log_{10} x - 12 = 0$ by Regula-Falsi Method. The root lies in the interval $(3,4)$.
7. Compute the root of equation $x^3 - 9x + 1 = 0$ using Bisection Method.

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Numerical Methods and its Applications

**Assignment 3: Fitting of Curves**

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1. The table gives the temperature $T$ (in ${}^0C$) and lengths $l$ (in mm) of heated rod. If $l = a_0 + a_1 T$, find the values of $a_0$ and $a_1$ using least squares method:

| $T$ | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|
| $l$ | 600.5 | 600.6 | 600.8 | 600.9 | 601.0 |

2. Find the best values of $a_0, a_1$ and $a_2$ so that the parabola $y = a_0 + a_1 x + a_2 x^2$ fits the data:

| x | 0.78 | 1.56 | 2.34 | 3.12 | 3.81 |
|---|---|---|---|---|---|
| y | 2.50 | 1.20 | 1.12 | 2.25 | 4.28 |

3. Determine the constants $a$ and $b$ by the least squares method such that $y = ae^{bx}$ fits the following data:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| y | 15.3 | 20.5 | 27.4 | 36.6 | 49.1 | 65.6 | 87.8 | 117.6 |

4. Fit a function of the form $y = ax^b$ to the following data:

| x | 2 | 4 | 7 | 10 | 20 | 40 | 60 | 80 |
|---|---|---|---|---|---|---|---|---|
| y | 43 | 25 | 18 | 13 | 8 | 5 | 3 | 2 |

5. Fit a linear function for the following data:

| x | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 3.8 | 3.7 | 4 | 3.9 | 4.3 | 4.2 | 4.2 | 4.4 | 4.5 | 4.5 |

6. Fit the curve $y = cx^d$ to the following data:

| x | 2.2 | 2.7 | 3.5 | 4.1 |
|---|---|---|---|---|
| y | 65 | 60 | 53 | 50 |

7. Find the best values of $c$ and $d$ if the curve $y = ce^{dx}$ is fitted to the following data:

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| y | 1.5 | 4.6 | 13.9 | 40.1 | 125.1 | 299.5 |

8. Find the best values of $a$, $b$ and $c$ so that $y = a + bx^2 + cx^3$ fits the following data

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| y | 1 | 0 | 3 | 10 | 21 | 36 | 55 | 78 |

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Numerical Methods and its Applications

**Assignment 4: Interpolation**

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1. Evaluate $\Delta^2 \cos(cx+d)$.
2. Evaluate $\Delta\left(\frac{2^x}{x+h}\right)$
3. Estimate the missing term in the following data:

| x | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| y | 1.4 | ? | 1.76 | 2.00 | 2.28 |

4. The following table gives the population (in lacs) of a town during the last six census. Estimate the increase in the population during the period 1922 to 1955.

| **Year** | 1911 | 1921 | 1931 | 1941 | 1951 | 1961 |
|---|---|---|---|---|---|---|
| **Population** | 12 | 15 | 20 | 27 | 39 | 52 |

5. Find the divided differences for the following data and hence obtain $f(15)$:

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

6. Find the interpolating polynomial for the data.

| $x$ | 0 | 1 | 2 | 5 |
|---|---|---|---|---|
| $f(x)$ | 2 | 3 | 12 | 147 |

7. Compute $f(2.45)$ from the following table:

| $x$ | 0·00 | 0·50 | 1·00 | 1·50 | 2·00 | 2·50 | 3·00 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0·000 | 0·191 | 0·341 | 0·433 | 0·477 | 0·494 | 0·499 |

8. Compute $f(5)$ using Newton's divided difference formula (Newton's general interpolation formula) from the given data:

| $x$ | 1 | 3 | 4 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 8 | 15 | 19 | 32 | 40 |

9. Estimate the missing terms in the following data:

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 0.00 | 0.21 | ? | 0.96 | 1.66 | 2.68 | ? | 5.98 | 8.40 |

10. Find the polynomial satisfied by $(0,1)$, $(1,2)$, $(2,11)$, $(3,34)$ using Newton's forward interpolation formula.
11. Find value of $y$ at $x = 9$ using Lagrange's interpolation formula, given that:

| x | 1 | 3 | 4 | 8 | 10 |
|---|---|---|---|---|---|
| y | 8 | 15 | 19 | 32 | 40 |

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Numerical Methods and its Applications

**Assignment 5: Numerical Differentiation and Integration**

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1. The following table gives the angular displacements $\theta$ (radians) at different intervals of time $t$ (seconds)

| $\theta$ | 0·052 | 0·105 | 0·168 | 0·242 | 0·327 | 0·408 | 0·489 |
|---|---|---|---|---|---|---|---|
| $t$ | 0 | 0·02 | 0·04 | 0·06 | 0·08 | 0·10 | 0·12 |

Calculate the angular velocity at the instant $t = 0.06$

2. Compute $f'(1\cdot16)$ and $f''(1\cdot16)$ from the following table

| $x$ | 1·11 | 1·12 | 1·13 | 1·14 | 1·15 | 1·16 |
|---|---|---|---|---|---|---|
| $f(x)$ | 6·2321 | 6·2544 | 6·2769 | 6·2996 | 6·3225 | 6·3456 |

3. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ using *Simpson's One-Third Rule*.

4.

5. Evaluate $\int_0^{\pi/4} \tan x\, dx$ using *Trapezoidal Rule*.

6. Find value of $\int_1^{2\cdot2} \ln x\, dx$ by *Simpson's Three-Eighth Rule*

7. Evaluate $\int_2^9 \sqrt{x^2+5}\, dx$ using *Trapezoidal Rule*.

8. The velocities of a car (running on single straight road) at intervals of 2 minutes are given below:

| Time in minutes | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Velocity in km/hr. | 0 | 22 | 30 | 27 | 18 | 7 | 0 |

Apply *Simpson's Rule* to find the distance covered by the car.

9. Calculate the area bounded by the curve using the following data:

| x | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|
| y | 23 | 19 | 14 | 11 | 12.5 | 16 | 19 | 20 | 20 |

10. A solid of revolution is formed by rotating about the X-axis , the area between the X-axis, the lines x=0 and x=1 and a curve through the points with the following co-ordinates:

| x | 0.00 | 0.25 | 0.50 | 0.75 | 1 |
|---|---|---|---|---|---|
| y | 1.00 | 0.9896 | 0.5589 | 0.9089 | 0.8415 |

Compute the volume of the solid formed. (The volume of the solid = $\pi \int_a^b y^2 dx$)

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Numerical Methods and its Applications

**Assignment 6: Numerical Solution of Differential Equations**

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1. Solve by Euler's method: the equation $\frac{dy}{dx} = xy$ with $y(0) = 1$ and find $y(0.04)$ by taking $h = 0.01$.
2. Find value of $y(0.6)$ using Euler's Modified method. Given differential equation $\frac{dy}{dx} = x + y$ with initial condition $y = 0$ when $x = 0$. Take $h = 0.2$.
3. Use Runge-Kutta second and fourth order method to find an approximate value of $y(2)$. Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial conditions $y(0) = 1$. Take $h = 1$.
4. Given that $\frac{dy}{dx} = y + x^2$ with $y(4) = 4$. Find $y(4.2)$ correct to four decimal places using Taylor's Method.
5. Given that $\frac{dy}{dx} - \sqrt{xy} = 2$ with $y(1) = 1$. Find $y(1.5)$ by Euler's method. Take $h = 0.5$.
6. Solve $\frac{dy}{dx} = 1 - y$ with $y(0) = 0$. Find $y(0.5)$ using Taylor's Method.
7. Generate $y(0.2)$, $y(0.4)$ using $4^{th}$ order Runge-Kutta formula for $\frac{dy}{dx} = -2y$ with $y(0) = 1$.
8. Determine $y(3)$ using $2^{nd}$ order Runge-Kutta formula. Given that $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(2) = 1$. Take $h = 0.5$.