Practical 2:Partial Derivatives and Gradient

1. $\phi = xyz$, $\bar{u} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ Find $\dfrac{\partial^3(\phi u)}{\partial x^2 \partial z}$ at $(1, 2, 1)$.

2. If $\bar{r} = e^{-\lambda x}\left(\bar{a}\sin\lambda y + \bar{b}\cos\lambda y\right)$ then prove that $\dfrac{\partial^2 \bar{r}}{\partial x^2} + \dfrac{\partial^2 \bar{r}}{\partial y^2} = 0.$

3. Prove that $\nabla f(r) = f'(r)\nabla r$. Hence find $\nabla(\sin^2 r)$.

4. $\bar{a} = \alpha x\,\hat{i} + \beta y\,\hat{j} + \gamma z\,\hat{k}$ prove that $\nabla(\bar{a}.\bar{r}) = 2\bar{a}$.

5. If $\nabla\phi = 2xyz^3\,\hat{i} + x^2 z^3\,\hat{j} + 3x^2 yz^3\,\hat{k}$ then find ϕ if $\phi(1, 1, 1) = 1$.