Practical 1:Vector Function of One Variable

1. If three mutually perpendicular unit vectors $\bar{a}, \bar{b}, \bar{c}$ are functions of t, then prove that
$$\frac{d\bar{a}}{dt} = \pm \left(\frac{d\bar{b}}{dt} \times \bar{c} + \bar{b} \times \frac{d\bar{c}}{dt} \right).$$

2. If $\bar{u} = (t^2 - 1)\,\hat{i} + \cos t\,\hat{j}, \bar{v} = \sin t\,\hat{i} + e^t\,\hat{j}$ then find $\lim\limits_{t \to 0}(\bar{u} \times \bar{v})$.

3. If $\bar{f} = \dfrac{\tan 4x}{x}\hat{i} + \dfrac{\log(1 + 2x)}{x}\hat{j} + \dfrac{3^x - 1}{x}\hat{k}, x \neq 0$ then find $\bar{f}(0)$ if $\bar{f}$ is continuous at $x = 0$.

4. Find the unit tangent vector to the curve $x = t^2, y = 2t, z = \frac{-1}{2}t^2$ at the point $t = -3$.

5. Find the acute angle between tangents to the curve $\bar{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ at $t = 1$ and $t = -1$.