Progressive Education Society's

Modern College of Arts, Science and Commerce Shivajinagar (Autonomous), Pune 411005

Department of Mathematics

S.Y.B.Sc.

Ordinary Differential Equations

Practical 1: Linear Differential Equation with constant coefficients

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Question: Find the solution of the following linear differential equations:

1. $(D+2)^3 y = 0$
2. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$
3. $\left(D^4 - 6D^3 + 18D^2 - 24D + 16\right) y = 0$
4. $\left(D^2 + 9\right) y = 0$
5. $\left(D^4 - D^3 - 18D^2 + 52D - 40\right) y = 0$
6. $\left(D^2 - 9\right) y = 3e^{2x} + 2e^{3x}$
7. $\left(D^3 - D^2 - 6D\right) y = 1 + x + e^{-x}$
8. $\left(D^2 - 4\right) y = 3x^2$
9. $\left(D^3 + 1\right) y = 4\cos^3 x$
10. $\left(D^2 - 2D + 5\right) y = e^{2x}\sin x$
11. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 9y = 40\sin 5x$
12. $\left(D^2 + 2)\right) y = x^2 e^{3x} + e^x \cos 2x$
13. $\left(D^2 - 2D + 1\right) y = xe^x + 7x - 2$
14. $\left(D^4 + 5D^2 + 6\right) y = \cos 2x + \sin 3x$
15. $(D-2)^2 y = \frac{1}{x^2} e^{2x}$
16. $\left(D^2 + D + 1\right) y = e^{4x}(2x+3)$
17. $\left(D^3 - D^2 + 3D + 5\right) y = e^x \cos 2x$
18. $\left(D^2 - 1\right) y = x\cos 3x$

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Practical 2: Non-homogeneous Linear Differential Equations

Question.1: Solve the following differential equations by method of variation of parameters:

1. $(D^2-1)y=\dfrac{2}{1+e^x}$
2. $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}=e^x\sin x$
3. $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-y=x^2e^x$
4. $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=x^3e^x$
5. $(x+2)\dfrac{d^2y}{dx^2}-(2x+5)\dfrac{dy}{dx}+2y=(x+1)e^x$
6. $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+9y=x^{-2}e^{2x}$
7. $y''-2y'+y=\dfrac{e^x}{x^2}$
8. $(x^2+1)y''-2xy'+2y=6(x^2+1)^2$

Question.2: Solve the following differential equations by method of reduction:

1. $(3-x)y''-(9-4x)y'+(6-3x)y=0$
2. $\dfrac{d^2y}{dx^2}-(1+x)\dfrac{dy}{dx}+xy=x$
3. $x\dfrac{d^2y}{dx^2}+(1-x)\dfrac{dy}{dx}-y=e^x$
4. $x\dfrac{d^2y}{dx^2}+(x-1)\dfrac{dy}{dx}-y=x^2$
5. $x\dfrac{d^2y}{dx^2}-(2x+1)\dfrac{dy}{dx}-(x+1)y=x^3e^x$
6. $x^2y''+y'-(1+x^2)y=e^{-x}$
7. $xy''+(x+2)y'-2y=x^3$
8. $(1-x^2)\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}-(1+x^2)y=x$
9. $y''-4xy'+4x^2y=e^{x^2}$

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Practical 3: Power Series Solutions

Question: Find the power series solution of the following differential equation

1. $\left(x^2-1\right)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-y=0$ near $x=0$.
2. $\left(1-x^2\right)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}-y=0$ near $x=0$.
3. $\left(x^2-1\right)\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+xy=0$ near $x=0$.
4. $y''-xy'+2y=0$ near $x=1$.
5. $\left(1-x^2\right)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}-y=0$ near $x=0$.
6. $\dfrac{d^2y}{dx^2}-2x^2\dfrac{dy}{dx}+4xy=x^2+2x+4$ in powers of x.
7. $\dfrac{d^2y}{dx^2}+x^2y=2+x+x^2$ about $x=0$.
8. $y''-y'=x$ about $x=0$.
9. $\left(x^2-1\right)y''+xy'-y=0$ near $x=0$.
10. $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+2y=0$ about $x=1$.

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Practical 4: System of first order equations

Question: Solve the following simultaneous equations:

1. $\frac{dx}{dt} = x - 2y,\ \frac{dy}{dt} = 5x + 3y.$
2. $\frac{dx}{dt} + 7x - y = 0,\ \frac{dy}{dt} + 2x + 5y = 0.$
3. $\frac{dx}{dt} + 2x - 3y = t,\ \frac{dy}{dt} - 3x + 2y = e^{2t}.$
4. $\frac{dx}{dt} + 2x + 4y = 1 + 4t,\ \frac{dy}{dt} - 3x + 2y = \frac{3t^2}{2}.$
5. $\frac{dx}{dt} + 5x + y = e^t,\ \frac{dy}{dt} - x + 3y = e^{2t}.$
6. $\frac{dx}{dt} - y = t,\ \frac{dy}{dt} + x = 1.$
7. $\frac{dx}{dt} + x - y = e^t,\ \frac{dy}{dt} + y - x = 0.$
8. $\frac{dz}{dx} + 4z + 3y = x,\ \frac{dy}{dx} + 2z + 5y = e^x.$
9. $\frac{dz}{dx} = x + y,\ \frac{dy}{dx} = x + z.$
10. $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,\ \frac{dy}{dt} + 5x + 3y = 0.$
11. $\frac{dx}{dt} - \frac{dy}{dt} - y = -e^t,\ \frac{dy}{dt} + x - y = e^{2t}.$
12. $\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t,\ \frac{dx}{dt} - \frac{dy}{dt} + 4x = 4\cos t - 3\sin t.$
13. $\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t,\ 3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^t.$
14. $4\frac{dx}{dt} - \frac{dy}{dt} + 3x = \sin t,\ \frac{dx}{dt} + y = \cos t.$

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Practical 5: Miscellaneous

1. Test the linear independence of the following sets of functions
   (a) $1+x, 1+2x, x^2$
   (b) $x^2-1, x^2-x+1, 3x^2-x-1$
   (c) $e^x, e^{-x}, \sin ax$
   (d) $\sin x, \cos x, \sin 2x$
2. Prove that $\sin 2x$ and $\cos 2x$ are solutions od differential equation $y'' + 4y = 0$ and these solutions are linearly independent.
3. Find the solution of differential equation $y''' - y'' - 4y' + 4y = 0$ and determine whether the solutions are linearly dependent or independent.
4. Solve the following simultaneous equations: $(5D+4)y - (2D+1)z = e^{-x}$, $(D+8)y - 3z = 5e^{-x}$
5. Show that $x = 0$ is an ordinary point of differential equation $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - xy = 0$. Hence, find the power series solution of it.
6. Verify that e^x and x are solution of homogeneous equations corresponding to $(1-x)y_2 + xy_1 - y = 2(x-1)^2 e^{-x}$, $0 < x < 1$. Thus, find its general solution by method of variation of parameters.
7. Find the solution of the linear differential equation $(D^3 - 5D^2 + 8D - 4)y = e^{2x} + 2e^x + 3e^{-x}$.
8. Solve the differential equations by method of reduction $xy'' + 2(x+1)y' + (x+2)y = (x-2)e^{2x}$
9. Find the solution of the linear differential equation $(D^3 - 3D^2 - 6D + 8)y = xe^{-3x}$
10. Solve the differential equations by method of variation of parameters $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = \dfrac{1}{x^2}$.

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