Progressive Education Society's
Modern College of Arts, Science and Commerce (Autonomous),
Shivajinagar, Pune-5
Department of Mathematics

Class : SYBSc (Semester IV) 2020-21
Subject : Mathematics Practical IV (19ScMatU404)
(Based on Linear Algebra (19ScMatU401))

Practical No. 1 : Vector Space and Subspace

1. Let $V = \mathbb{R}^+$. For $x, y \in V$ and for $\alpha \in \mathbb{R}$, define $x + y = x \cdot y$ and $\alpha x = x^\alpha$. Show that V is a vector space over $\mathbb{R}$.

2. Show that $W = \{f | f(3) = 0\}$ is a subspace of the vector space V of all real valued functions.

3. Write the matrix $\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as the linear combination of the matrices $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$.

4. Show that the vectors $(1, 1, 1), (0, 1, 1), (0, 1, -1)$ generate $\mathbb{R}^3$.

5. Determine whether the matrices $\begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{bmatrix}$ have the same column space.

6. Let $U = \{(a, b, c) | a = b = c\}$ and $W = \{(0, b, c)\}$ be the subspaces of $\mathbb{R}^3$. Show that $\mathbb{R}^3 = U \oplus W$.

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Practical No. 2 : Basis and Dimension of Vector Space

1. Determine whether or not the vectors $(1,-2,1),(2,1,-1),(7,-4,1)$ in $\mathbb{R}^3$ are linearly dependent.

2. Show that the vectors $(1,1,1),(1,2,3),(2,-1,1)$ form a basis of $\mathbb{R}^3$.

3. Find the dimension and the basis of the solution space W of the system

$$\begin{aligned} x+2y+2z-r+3s&=0\\ x+2y+3z+r+s&=0\\ 3x+6y+8z+r+5s&=0 \end{aligned}$$

4. Find the coordinate vector of $v=(4,-3,2)$ relative to the basis $\{(1,1,1),(1,1,0),(1,0,0)\}$ of $\mathbb{R}^3$.

5. Find the rank and the nullity of a matrix $\begin{bmatrix} 1 & 2 & -3\\ 2 & 1 & 0\\ -2 & -1 & 3\\ -1 & 4 & -2 \end{bmatrix}$.

6. Let W be the space generated by the polynomials $u=t^3+2t^2-2t+1$, $v=t^3+3t^2-t+4$ and $w=2t^3+t^2-7t-7$.
   Find a basis and the dimension of W.

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Practical No. 3 : Linear Transformation I

1. Show that a map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x - y, 2x + y)$ is a linear transformation.

2. Determine whether a map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, y^2)$ is a linear transformation.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}$ be the linear mapping for which $T(1, 1) = 3$ and $T(0, 1) = -2$. Find $T(x, y)$ and hence compute $T(2, 3)$.

4. Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^3$ be linear transformations defined by $T_1(x, y) = (x + y, y)$ and $T_2(x, y) = (2x, y, x + y)$ respectively.
   Find $T_1 \circ T_1$ and $T_2 \circ T_1$. Is it possible to find $T_1 \circ T_2$?

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x + y, y + z, z + x)$.
   Show that T is a linear isomorphism. Hence find T^{-1}.

6. Show that a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (y, x+y-z)$ is surjective but not injective. Also find kernel of T.

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Practical No. 4 : Linear Transformation II

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by
$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.
Find a basis and the dimension of (i) Range of T, and (ii) Kernel of T.

2. Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & -1 & 2 & -1 \\ 1 & -3 & 2 & -2 \end{bmatrix}$ be the matrix of a linear map $T : \mathbb{R}^4 \to \mathbb{R}^3$.
Find the image and the kernel of T. Also verify the dimension theorem for T.

3. Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$.
Find the matrix of T in the basis $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator defined by $T(x, y) = (2x - 3y, x + 4y)$.
Find the matrix of T in the bases $\{(1, 0), (0, 1)\}$ and $\{(1, 3), (2, 5)\}$.

5. Let $E = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $F = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ be the bases of $\mathbb{R}^3$. Then

   (a) Find the transition matrix P from E to F.

   (b) Find the transition matrix Q from F to E. Is $Q = P^{-1}$?

   (c) If $v = (1, 2, 3)$ then show that $[v]_F = Q[v]_E$. Is this true for any vector $v \in \mathbb{R}^3$?

   (d) If A and B are the matrices of T defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$ with respect to E and F respectively, then verify that $B = P^{-1}AP$.

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Practical No. 5 : Eigenvalues and Eigenvectors

1. Find the characteristic polynomial, the eigenvalues, the eigenvectors and the eigenspaces corresponding to each eigenvalue of the matrix $\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$.

2. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

4. Determine whether the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$ is diagonalizable.

5. Find all eigenvalues and a basis of each eigenspace of the operator T on $\mathbb{R}^3$ defined by $T(x, y, z) = (2x + y, y - z, 2y + 4z)$.

6. Find the minimum polynomial of the matrix $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$.

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Practical No. 6 : Inner Product Space

1. Verify that the following is an inner product in $\mathbb{R}^2$.
   $< u, v >= x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$, where $u = (x_1, x_2), v = (y_1, y_2)$.

2. Find norm of each and the distance between $u = (1, 1, -1)$ and $v = (-1, 1, 0)$ with respect to the usual Euclidean dot product of $\mathbb{R}^3$.
   Also verify the parallelogram law : $||u + v|| + ||u - v|| = 2||u|| + 2||v||$.

3. Let V be the vector space of polynomials with inner product given by $< f, g >= \int_0^1 f(t)g(t)dt$. Let $f(t) = t + 2$ and $g(t) = t^2 - 2t - 3$.
   Find $< f, g >$, $||f||$ and $||g||$.

4. Let $u = (1, -2, 2), v = (0, -3, 4) \in \mathbb{R}^3$. Then find
   (a) An angle between u and v.
   (b) A unit vector orthogonal to u and v.
   (c) An orthogonal projection of u along v.

5. Let W be the subspace of $\mathbb{R}^5$ spanned by $(1, 2, 3, -1, 2)$ and $(2, 4, 7, 2, -1)$.
   Find a basis of the orthogonal complement $W^\perp$ of W.

6. Using Gram-Schmidt orthogonalization process, transform the basis
   (a) $\{(1, -3), (2, 2)\}$ to an orthonormal basis of $\mathbb{R}^2$.
   (b) $\{(1, 1, 1), (-1, 1, 0), (1, 2, 1)\}$ to an orthonormal basis of $\mathbb{R}^3$.
   (c) $\{(1, i, 0), (1, 2, 1 - i)\}$ of the subspace W of $\mathbb{C}^3$ to an orthonormal basis of W.