Progressive Education Society's
Modern College of Arts, Science and Commerce (Autonomous),
Shivajinagar, Pune-5
Department of Mathematics

Class : FYBSc (Semester II) 2020-21
Subject : Mathematics Practical II (19ScMatU203)
(Based on Integral Calculus (19ScMatU202))

Practical No. 7 : Integration

Evaluate the following integrals.

1. $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$
2. $\int \frac{3x+4}{(x+2)^2(x-6)}dx$
3. $\int \frac{x^2}{x^4-x^2-12}dx$
4. $\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)}dx$
5. $\int \frac{1}{(1+e^x)(1+e^{-x})}dx$
6. $\int \frac{\log x}{x(1+\log x)(2+\log x)}dx$
7. $\int \frac{x^2-1}{x^4-x^2+1}dx$

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Practical No. 8 : Reduction formulae

I) Evaluate the following integrals.

1. $\int \frac{7x+3}{\sqrt{3+2x-x^2}}dx$
2. $\int (3x+2)\sqrt{x^2+3x+1}dx$
3. $\int \frac{x^3+4x^2-6x+3}{\sqrt{5+6x-x^2}}dx$

II) Evaluate the following definite integrals.

1. $\int_0^{\pi/2} \sin^4 x \cos^8 x dx$
2. $\int_0^{\pi/6} \cos^2(6x) \sin^4(3x) dx$
3. $\int_0^4 x^{1/2}(4-x)^{3/2} dx$

III) If $I_n = \int \frac{1}{(x^2+1)^n}dx$ then prove that $2(n-1)I_n = x(x^2+1)^{1-n} + (2n-3)I_{n-1}$

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Practical No. 9 : Preliminaries of differential equations

1. Determine the order and degree of each of the following differential equation.

   (a) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$

   (b) $\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

   (c) $\frac{d^3y}{dx^3} + \sin\left(\frac{dy}{dx}\right) = 1$

2. Which of the following are homogeneous functions?
   Determine the degree in case they are homogeneous.

   (a) $f(x,y) = 2x^4 + 3x^2y^2 - 5xy^3$

   (b) $f(x,y) = (x^3 + x^2y)^3$

   (c) $f(x,y) = \frac{x^2+y^2}{xy-x^2}$

   (d) $f(x,y,z) = 3x^4 + 2x^2yz - 4xy^2z$

   (e) $u = \sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$

3. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ if

   (a) $u = \log(x^2+y^2)$

   (b) $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

4. If $u = x^y + y^x$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

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Practical No. 10 : Solution of differential equations of first order and first degree - I

1. Find the differential equation of the family of all straight lines making equal intercepts on the coordinate axes.

2. Find the differential equation of the family of curves $y = e^x(a\cos x + b\sin x)$, where a and b are arbitrary constants.

3. Find the differential equation of the family of circles $(x-h)^2 + (y-k)^2 = 1$, where h and k are arbitrary constants.

4. Solve the following differential equations.

   (a) $(xy^2 + x)dx + (yx^2 + y)dy = 0$

   (b) $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

   (c) $x^2ydx - (x^3 + y^3)dy = 0$

   (d) $(y\sin\frac{y}{x} - x)dx - x\sin\frac{y}{x}dy = 0$

   (e) $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$

   (f) $(3y - 7x - 3)dx + (7y - 3x - 7)dy = 0$

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Practical No. 11 : Solution of differential equations of first order and first degree - II

1. Solve the linear differential equation $\frac{dy}{dx} + y \sec x = \tan x$.
2. Solve Bernoulli's differential equation $(x^3y^2 + xy)dx - dy = 0$.
3. Check whether the following differential equations are exact and hence solve them.
   (a) $(y + \frac{y}{x} + \sin y)dx + (x + \log x + x\cos y)dy = 0$
   (b) $(x^4 + y^4)dx - xy^3dy = 0$
   (c) $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$
   (d) $(x^3 + xy^4)dx + 2y^3dy = 0$
   (e) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
4. Find the orthogonal trajectories of the family of parabolas $y = ax^2$.
5. Show that the family $y^2 = 4a(x + a)$ is self orthogonal.

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Practical No. 12 : Differential equations of first order and higher degree

1. Solve the following differential equations for p, where $p = \frac{dy}{dx}$.
   (a) $xyp^2 - (x^2 + y^2)p + xy = 0$
   (b) $p(p - y) = x(x + y)$
2. Solve the following differential equations for y.
   (a) $y = 2px + p^4x^2$
   (b) $e^{p-y} = p^2 - 1$
3. Solve the following differential equations for x.
   (a) $p^3 = y^4(y + xp)$
   (b) $p = \tan\left(x - \frac{p}{1+p^2}\right)$
4. Solve Lagrange's differential equation $y = (1 + p)x + p^2$.
5. Check whether the differential equation $(1 + p^2)(y - px) = 3p$ is in Clairaut's form. If yes, solve it.